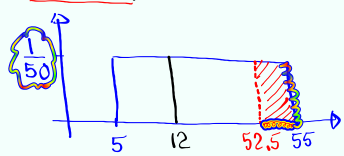
Statistics

Lecture 10

Feb 19-8:47 AM

Consider a uniform Prob. dist. for all Values from 5 to 55.

$5 \leq x \leq 55$



1) $P(x=12) = \boxed{0}$
Line

2) $P(x > 52.5)$

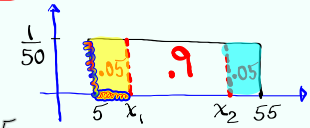
$= (55 - 52.5) \cdot \frac{1}{50}$

$= \frac{2.5}{50} = \boxed{.05} = \boxed{\frac{1}{20}}$

3) Find two Values that Separate the middle 90% from the rest.

$1 - .9 = .1$

$.1 \div 2 = .05$



$(x_1 - 5) \cdot \frac{1}{50} = .05$

$x_1 - 5 = 50(.05)$

$x_1 = 5 + 2.5$

$\boxed{x_1 = 7.5}$

$(55 - x_2) \cdot \frac{1}{50} = .05$

$55 - x_2 = 50(.05)$

$55 - 2.5 = x_2$ $\boxed{x_2 = 52.5}$

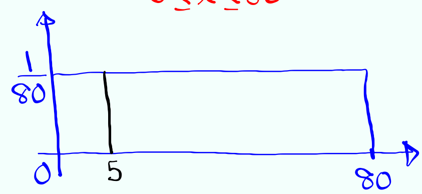
Oct 31-11:36 AM

wait time at Local DMV has a uniform Prob. dist and does not exceed 80 minutes.

$0 \le x \le 80$

P(wait exactly 5 mins)

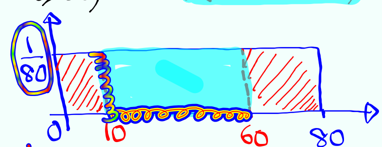
$= P(x=5) = \boxed{0}$



P(wait time is below 10 mins or above 60 mins.)

Total Area

$P(x<10 \text{ OR } x>60) = 1 - P(10<x<60)$

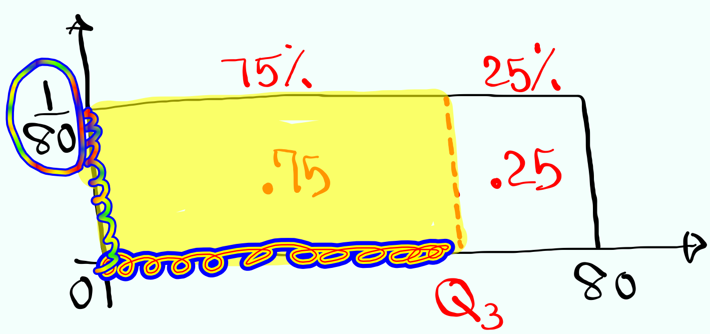


$= 1 - (60-10)\cdot \frac{1}{80}$

$= 1 - \frac{50}{80} = 1 - \frac{5}{8} = \boxed{\frac{3}{8}} = \boxed{.375}$

Oct 31-11:47 AM

Find Q_3 of wait time at that DMV.



$(Q_3 - 0)\cdot \frac{1}{80} = .75$

$Q_3 = 80(.75) = \boxed{60}$

Oct 31-11:55 AM

Standard Normal Prob. dist.: (SG 17)

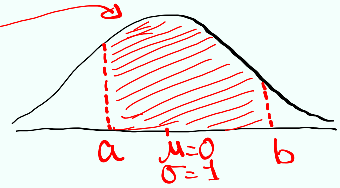
1) use Z, $P(Z=c)=0$

2) Dist. is Symmetric, Bell-shape with total Area $=1$.

3) Mean = Mode = Median

4) $\mu=0$, $\sigma=1$

$\boxed{P(a<Z<b)}$ is the Corresponding area within the bell-shape Graph.



How to find it:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

normalcdf(L, U, μ, σ)

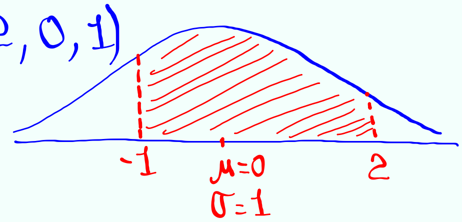
Oct 31-11:59 AM

Find $P(-1<Z<2)$

$=$ normalcdf$(-1, 2, 0, 1)$

$(-)$

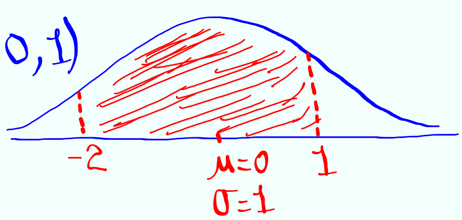
$\boxed{9}$ $\boxed{7}$



$= \boxed{.819}$

Find $P(-2<Z<1)$

$=$ normalcdf$(-2, 1, 0, 1)$



$= \boxed{.819}$

$(-)$

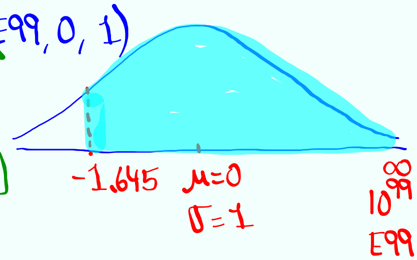
Oct 31-12:05 PM

Find $P(Z > -1.645)$

$= \text{normalcdf}(-1.645, E99, 0, 1)$

(−)

$\boxed{2nd}\ \boxed{,}$



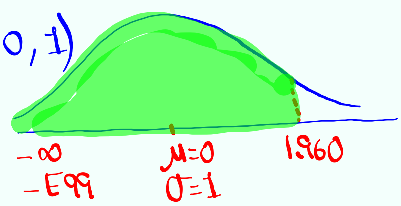
$= \boxed{.950}$

Find $P(Z < 1.960)$

$= \text{normalcdf}(-E99, 1.960, 0, 1)$

(−)

$\boxed{2nd}\ \boxed{,}$



$= \boxed{.975}$

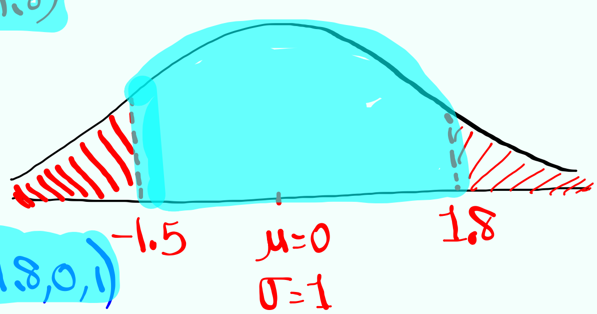
Oct 31-12:11 PM

$P(Z < -1.5 \text{ OR } Z > 1.8)$ → If it was AND $\boxed{0}$

$= 1 - P(-1.5 < Z < 1.8)$

↑

Total Area (Prob.)



$= 1 - \text{normalcdf}(-1.5, 1.8, 0, 1)$

$= \boxed{.103}$

Oct 31-12:18 PM

Doing Reverse

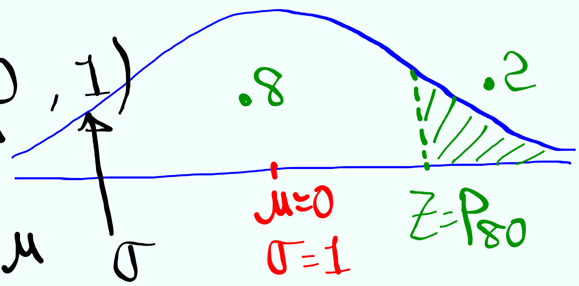
Find $Z = P_{80}$

80% below (Left)

$\boxed{2nd}\boxed{VARS}$ 20% above (Right)

$Z = \text{invNorm}(.8, 0, 1)$

left Area μ σ



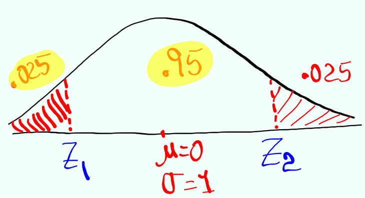
$= \boxed{.842}$

Oct 31-12:23 PM

Find two Z-Values that Separate the middle 95% from the rest.

$1 - .95 = .05$

$.05 \div 2 = .025$



$Z_1 = \text{invNorm}(.025, 0, 1) = \boxed{-1.960}$

$Z_2 = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

If $-2 \le Z \le 2$ → usual data element

Usual Range → 95% Range

$P(-1.960 < Z < 1.960) = .95$

SG 17 ✓

Oct 31-12:27 PM

Normal Prob. Dist.: (SG 18)

1) Use x, $P(x=c)=0$

2) data dist. is symmetric, bell-shape with total area $=1$. $N(\mu,\sigma)$

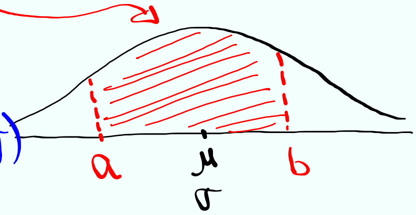
3) Mean = Mode = Median

4) μ & σ are given in the problem

$\boxed{P(a<x<b)}$ is the Corresponding area within the normal curve.

Using TI

normalcdf(L, U, μ, σ)



Oct 31-12:45 PM

Given $N(82, 6)$

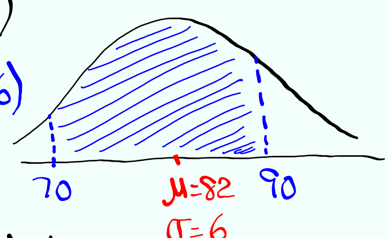
Normal Prob. Dist.

μ σ

Find $P(70<x<90)$

$=$ normalcdf$(70, 90, 82, 6)$

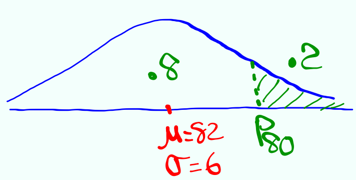
$= \boxed{.886}$



Find $x=P_{80}$, Round to whole #

$x=$ invNorm$(.8, 82, 6)$

$\approx \boxed{87}$



Oct 31-12:51 PM

Ages of teachers in LA County has a normal dist. with mean of 48 yrs and standard dev. of 8 yrs. $N(48,8)$

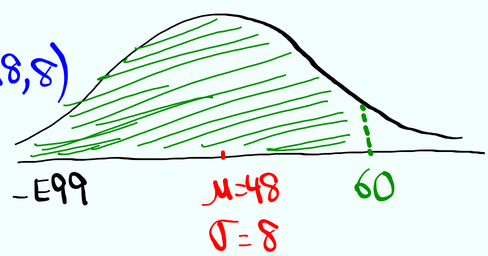
If one teacher is randomly selected, find the Prob. that his/her age is below 60.

$P(x<60)$

$= \text{normalcdf}(-E99, 60, 48, 8)$

(−) $\boxed{2nd}\boxed{,}$

$= \boxed{.933}$

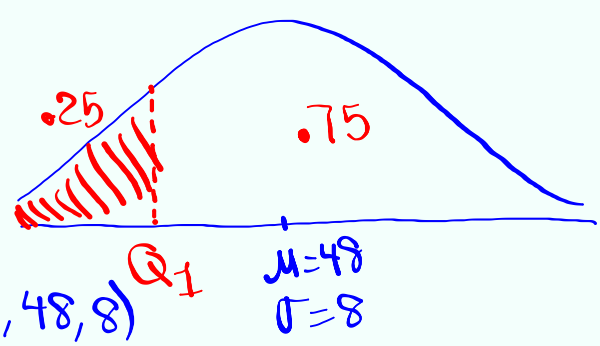


Oct 31-12:57 PM

Find Q_1 for ages of teachers in LA County.

25% below

75% above



$Q_1 = \text{invNorm}(.25, 48, 8)$

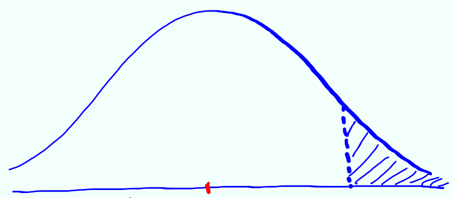
$\approx \boxed{43}$

Oct 31-1:03 PM

Scores on SAT are normally dist with mean of 1250 and standard dev. of 100. $N(1250, 100)$

If one SAT exam is randomly selected, find the Prob. that it is higher than 1475.

$P(x > 1475)$

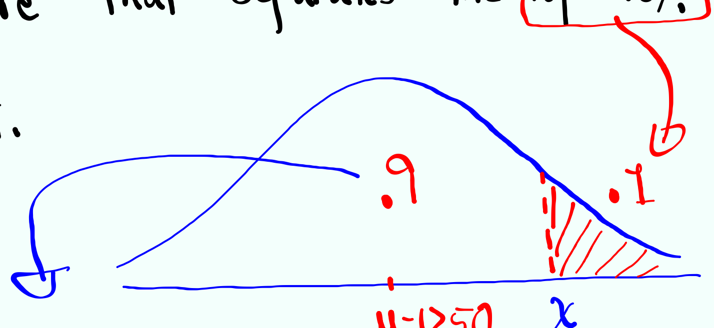


$= \text{normalcdf}(1475, E99, 1250, 100)$ $\mu = 1250$ 1475

$= \boxed{.012}$ $\sigma = 100$

Oct 31-1:06 PM

Find the Score that Separates the top 10% from the rest.



$x = \text{invNorm}(.9, 1250, 100)$ $\mu = 1250$

$\sigma = 100$

$\approx \boxed{1378}$

Oct 31-1:11 PM

Ages of students are N.D. with

$\mu = 30$ & $\sigma = 5$. $N(30, 5)$

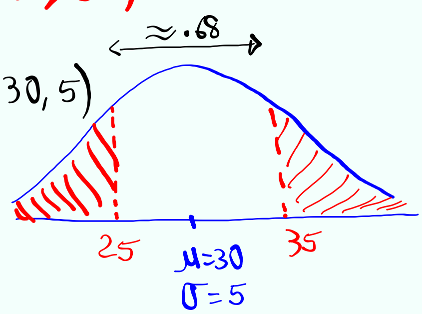
If one student is randomly selected,

find the Prob. his/her age is more than 35 or below 25.

$P(x < 25 \text{ or } x > 35)$

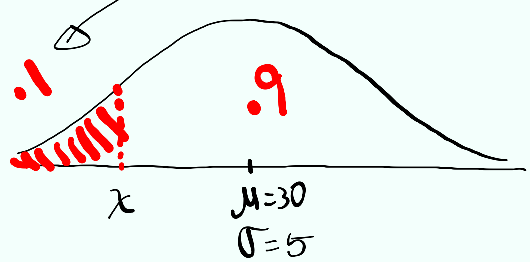
$= 1 - \text{normalcdf}(25, 35, 30, 5)$

$\approx \boxed{.317} \approx 32\%$



Oct 31-1:14 PM

Find the age that separates the bottom 10% from the rest.



$x = \text{invNorm}(.1, 30, 5) \approx \boxed{24}$

SG 18 ✓

Oct 31-1:20 PM

Central Limit Theorem (SG 19)

Clear all lists

Store 2, 4, 6, and 8 in L1.

Use $\boxed{\text{1-Var Stats}}$ with L1 only to find

$\mu = \bar{x} = 5$ $\quad \sigma = \sigma_x = 2.236$ $\quad \sigma^2 = \sigma_x^2 = 5$

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4: \sigma x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

Take all Samples of Size 2 with replacement from 2,4,6,and 8.

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

16 Samples of Size 2.

Oct 31-1:34 PM

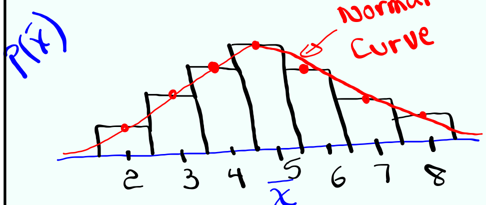
Find $\bar{x}$ of each Sample

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 means

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

Draw Prob. dist. Hist.



$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3

$\mu_{\bar{x}} = 5$ $\quad \sigma_{\bar{x}} = 1.581$ $\quad \sigma_{\bar{x}}^2 = 2.5 = \frac{5}{2}$

Oct 31-1:41 PM

Clear all lists

Store 2, 4, 6, 8, and 10 in L1.

Use $\boxed{\text{1 - Var Stats}}$ with L1 only to find

$\mu = 6$ $\sigma = 2.828$ $\sigma^2 = 8$

take all Samples of Size 2 with replacement from 2, 4, 6, 8, and 10.

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

25 Samples of Size 2

Oct 31-1:52 PM

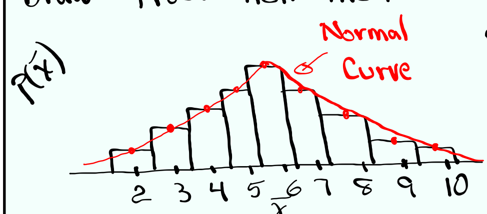
Find $\bar{x}$ of each Sample

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

25 means

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25

Draw Prob. dist. Hist.



$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use 1-Var Stats with L2 & L3

$\mu_{\bar{x}} = 6$ $\sigma_{\bar{x}} = 2$ $\sigma_{\bar{x}}^2 = 4 = \frac{8}{2}$

Oct 31-1:57 PM

Central Limit Theorem

$\boxed{\mu_{\bar{x}} = \mu}$

$\boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}}$

$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

ex: Consider a normal Prob. dist. with $\mu = 35$ and $\sigma = 6$

If we take all Sample of Size 4,

$\mu_{\bar{x}} = \mu = \boxed{35}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = \frac{6}{2} = \boxed{3}$

Oct 31-2:07 PM

Ages of students are normally dist. with $\boxed{\mu = 32}$ and $\boxed{\sigma = 6.}$

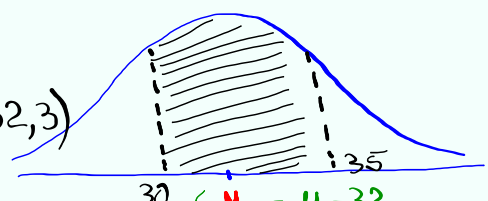
If we take Samples of $\boxed{4 \text{ Students}}$, $\boxed{n = 4}$

Find the prob. that $\boxed{\text{their mean age}}$ ($\bar{x}$) is $\boxed{\text{between } 30 \text{ \& } 35.}$

$P(30 < \bar{x} < 35)$

$= \text{normalcdf}(30, 35, 32, 3)$

$= \boxed{.589}$

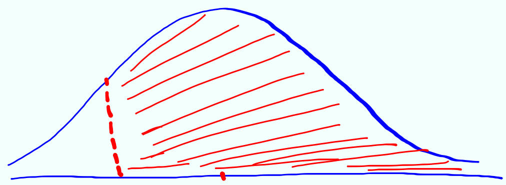


CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = \frac{6}{2} = 3 \end{cases}$

Oct 31-2:11 PM

Exam Scores are normally dist. with $\sigma = 10$
$\mu = 86$ ✓
mean of 86 and Standard dev. of 10.
$n = 2$
If we randomly select 2 exams
$\bar{x}$
find the prob. that their mean is above 80.

$P(\bar{x} > 80)$



$= \text{normalcdf}(80, E99, 86, 10/\sqrt{2})$

$= \boxed{.802}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 86 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{2}} \end{cases}$

Oct 31-2:16 PM

find $\bar{x} = P_{95}$ for randomly selected 5 exams.

$\bar{x} = \text{invNorm}(.95, 86, 10/\sqrt{5})$

$\approx \boxed{93}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 86 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{5}} \end{cases}$

SG 19 ✓

Oct 31-2:23 PM